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RECEPTIVITY OF THE BOUNDARY LAYER ON A FLAT PLATE WITH A BLUNTED LEADING EDGE TO STEADY NONUNIFORMITY OF THE FREE STREAM

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The flow past a flat plate with a blunted leading edge by a flow of a viscous incompressible fluid with a small spanwise-periodic, steady nonuniformity of the velocity profile is considered. Such a flow simulates the interaction of one type of vortex disturbances of a turbulent external flow with the boundary layer. The solution obtained predicts generation of strong disturbances in the boundary layer, which are similar to the streaky structure observed in the case of high free-stream turbulence. It is shown that the boundary-layer flow on blunted bodies is more sensitive to vortex disturbances than on a plate with a sharp leading edge.

Introduction. In the case of a high level of free-stream turbulence ($0.1\% < \varepsilon_t < 5\%$), the laminar–turbulent transition occurs without formation of the Tollmien–Schlichting waves [1]. Instead of them, the growth of low-frequency perturbations of velocity is observed. Flow visualization shows that these perturbations are narrow streaks extended in the streamwise direction [2]. It is assumed that these streaky structures appear as a result of penetration of vortices from the external flow into the boundary layer and their subsequent amplification in it. Therefore, the solution of the problem of receptivity of the boundary layer to vortex disturbances is an important component in developing the theory of the laminar–turbulent transition in the case of an elevated level of free-stream turbulence.

This problem has been solved only for the particular case of interaction of streamwise vortices with the boundary layer on a flat plate [3, 4]. This is the simplest case, since the free-stream vorticity field is not distorted by the flow near the leading edge. However, such a deformation involves additional amplification of perturbations due to the expansion of vortex filaments [5]. The greatest amplification is experienced by perturbations whose vortex lines intersect the leading edge. Hence, these perturbations (and not the streamwise vortices) should generate the streaky structure most effectively. It is shown in [5] that vorticity perpendicular to the leading edge (or nonuniformity of the velocity profile in the spanwise direction) can even lead to a local separation of the boundary layer. The analysis [5] was made for large-scale disturbances of small but finite amplitude. Under the assumptions accepted, the development of disturbances is actually inviscid, and the governing influence is exerted by nonlinear effects. However, it follows from the experimental results of Westin et al. [6] that the transverse size of the streaky structure is small, and viscosity plays a significant role in the development of this structure. In addition, the amplitude of perturbations observed in [6] is small for manifestation of strong nonlinear effects. In the present work, the problem of interaction of a nonuniform flow with the boundary layer is solved under the following assumptions: the characteristic size of disturbances is assumed to be of the order of the boundary-layer thickness and the evolution of disturbances is linear in terms of their amplitude.

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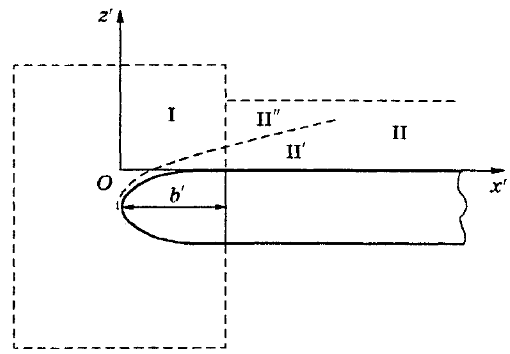

Fig. 1

1. Basic Assumptions. We consider a viscous incompressible fluid flow past a flat plate with a blunted leading edge. The characteristic size of the leading-edge bluntness is denoted by b'. (Hereinafter the dimensional quantities are primed, and the dimensionless quantities are not.) To describe the flow, we introduce a Cartesian coordinate system whose axes Ox, Oy, and Oz are directed along the flow, parallel to the leading edge, and normal to the plate surface. The origin is located in such a way that the plane xOy coincides with the upper surface of the plate, and the leading edge corresponds to $x' = 0$ (Fig. 1). The free stream is assumed to be weakly nonuniform in the spanwise direction. The streamwise component of velocity u' in the free stream is given by

$$u' = u'_\infty(1 + \varepsilon\, e^{-\sigma' x'} \cos(2\pi y'/\lambda')), \tag{1.1}$$

where u'_∞ is the free-stream velocity, λ' is the period of nonuniformity, ε is the small parameter equal to the amplitude of nonuniformity for $x' = 0$, the parameter $\sigma' = 4\pi^2\nu'/(u'_\infty\lambda'^2)$ describes the damping of the nonuniformity due to the action of viscosity, and ν' is the kinematic viscosity. The transverse (v') and vertical (w') components of the free-stream velocity are equal to zero. In addition, the Reynolds number based on the period of nonuniformity $\mathrm{Re} = u_\infty\lambda'/\nu'$ is assumed to be rather large. Then the expression for the free-stream velocity (1.1) is the solution of the Navier–Stokes equations with accuracy to small quantities of order ε/Re. To simplify the problem, we assume that the period of nonuniformity λ' is small as compared to the bluntness radius b'. In contrast to [5], we confine ourselves to solving the linear (in terms of ε) problem on a weakly nonuniform flow past a flat plate.

The velocity components and the pressure p' are represented as

$$u' = u'_\infty[U_b(x', z') + \varepsilon u(x', z')\cos(2\pi y'/\lambda')], \quad v' = u'_\infty[\varepsilon v(x', z')\sin(2\pi y'/\lambda')],$$

$$w' = u'_\infty[W_b(x', z') + \varepsilon w(x', z')\cos(2\pi y'/\lambda')], \quad p' = \rho' u'^2_\infty[P_b(x', z') + \varepsilon p(x', z')\cos(2\pi y'/\lambda')],$$

where ρ' is the density; the dimensionless velocity components U_b and W_b and the pressure P_b correspond to a uniform flow past a flat plate, whereas u, v, w, and p correspond to perturbations generated by nonuniformity. The evolution of perturbations is described by the Navier–Stokes equations linearized relative to the main flow with no-slip boundary conditions at the plate surface. The free-stream boundary conditions follow from (1.1): $u \to e^{-\sigma' x'}$ and $v, w \to 0$ as $x' \to -\infty$.

2. Solution at the Initial Section ($x'/\lambda' \ll \mathrm{Re}$). We seek the solution of the problem by the method of matching of asymptotic solutions. The flow field is divided into two regions shown schematically in Fig. 1. Region I is the vicinity of the leading edge with the characteristic size b' ($x' \approx b'$ and $z' \approx b'$). The flow in this region is inviscid outside a thin boundary layer and is described by linearized Euler equations with no-slip conditions on the wall. Because of the small size of region I, we may ignore the lengthwise damping of nonuniformity and consider the free-stream boundary conditions in the form $u \to 1$ and $v, w \to 0$ as $x'/b' \to -\infty$.

The problem in region I in a similar formulation was solved in [5]. In what follows, we need only the asymptotic behavior of this solution near the wall and at a large distance downstream of the leading edge:

$$\frac{x'}{b'} \to \infty, \quad \frac{z'}{b'} \to 0: \quad u \to 1, \quad v \to -\frac{2\pi}{a}\frac{b'}{\lambda'} \ln\Big(\frac{z'}{b'}\Big), \quad w \to -\frac{4\pi^2}{a}\Big(\frac{b'}{\lambda'}\Big)^2 \frac{z'}{b'} \ln\Big(\frac{z'}{b'}\Big).$$

The constant $a \approx 1$ in the expression for v depends on the shape of leading-edge bluntness.

Viscous terms become significant in region II of length $x' \approx \lambda'\mathrm{Re}$. The vertical size of this region $z' \approx \lambda'$ is determined by the distance from the wall at which the displacing action of the boundary layer on velocity perturbations is manifested. In region II, we introduce the dimensionless coordinates $X = x'/(\lambda'\mathrm{Re})$ and $Z = z'/\lambda'$. For $X \ll 1$, we can identify two subregions (Fig. 1): subregion II′ (boundary layer near the wall in which $Z \sim \sqrt{X}$ and $z' \approx \sqrt{\nu x'/u'_\infty}$ and the flow is viscous) and subregion II″ (inviscid subregion in which $Z \gg \sqrt{X}$ and $z' \approx \lambda'$). The boundary conditions for $X = 0$ in inviscid subregion II″ are obtained by matching with the asymptotic solution for $x' \to \infty$ in the vicinity of the leading edge:

$$u(0, z) = 1, \quad v(0, z) = A, \quad w(0, z) = -2\pi A Z. \tag{2.1}$$

Here $A = (2\pi b'/(\lambda' a)) \ln(b'/\lambda') \gg 1$. We note that it was assumed that $b'/\lambda' \gg 1$ in deriving the condition for v. The boundary conditions for $X = 0$ in the boundary layer (subregion II′) should be found from the solution of the boundary-layer problem in region I. However, a certain solution will be found there, which agrees with the boundary conditions in subregion II″ for $X \to 0$. This approach is justified if we assume that the perturbations introduced into the boundary layer in the vicinity of the leading edge decay at a distance of order b' from it.

Because of the linearity, the solution of the problem for velocity perturbations in region II may be represented as a sum of the solutions of two problems: (1) with nonzero conditions for u and zero conditions for v and w for $X = 0$; (2) with zero conditions for u and nonzero conditions for the remaining components of velocity. The solution of the first problem describes the decay of the initial nonuniformity of the velocity profile due to viscous dissipation. The value of velocity perturbations remains of the order of unity for all X. The solution of the second problem, as will be shown below, describes the increase in the streamwise component of velocity up to a value of order Re for $X \approx \mathrm{Re}$. For large X, the total perturbations are determined by solving the latter problem with nonzero conditions for v and w. This problem is considered below. Its solution in region II is sought in the form

$$u = A\,\mathrm{Re}\,U(X, Z), \quad v = AV(X, Z), \quad w = AW(X, Z), \quad p = (A/\mathrm{Re})P(X, Z), \tag{2.2}$$

where the functions U, V, W, and P are universal, i.e., independent of the shape of the leading edge, Re, and other parameters. Substituting these expressions into the linearized Navier–Stokes equations and rejecting the terms of order $1/\mathrm{Re}^2$, we obtain the following system:

$$U_0 \frac{\partial U}{\partial X} + \frac{\partial U_0}{\partial X} U + W_0 \frac{\partial U}{\partial Z} + \frac{\partial U_0}{\partial Z} W = \frac{\partial^2 U}{\partial Z^2} - 4\pi^2 U,$$

$$U_0 \frac{\partial V}{\partial X} + W_0 \frac{\partial V}{\partial Z} = -2\pi P + \frac{\partial^2 V}{\partial Z^2} - 4\pi^2 V, \tag{2.3}$$

$$U_0 \frac{\partial W}{\partial X} + \frac{\partial W_0}{\partial X} U + W_0 \frac{\partial W}{\partial Z} + \frac{\partial W_0}{\partial Z} W = -\frac{\partial P}{\partial Z} + \frac{\partial^2 W}{\partial Z^2} - 4\pi^2 W, \quad \frac{\partial U}{\partial X} + 2\pi V + \frac{\partial W}{\partial Z} = 0.$$

In deriving (2.3), we assumed that the main flow in region II corresponds to the Blasius boundary layer

$$U_b = U_0(\eta), \quad W_b = \frac{1}{\mathrm{Re}} W_0(\eta), \quad P_b = O\Big(\frac{1}{\mathrm{Re}^2}\Big), \quad U_0 = f', \quad W_0 = \frac{1}{2\sqrt{X}}(\eta f' - f), \quad \eta = \frac{Z}{\sqrt{X}},$$

where the function f is found from the boundary-value problem

$$f''' + (1/2) f f'' = 0, \qquad f(0) = f'(0) = 0, \quad f'(\infty) = 1.$$

We consider the solution of (2.3) in subregion II″. In this subregion, because of the simple form of the main flow [$U_b = 1$, $W_b = W_{00}/(\mathrm{Re}\sqrt{X})$, and $W_{00} = (1/2) \lim_{\eta \to \infty} (\eta f' - f)$] the equations of motion are significantly simplified and acquire the following form:

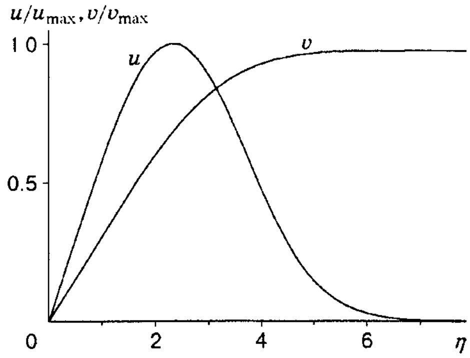

Fig. 2

$$\frac{\partial U}{\partial X}+\frac{W_{00}}{\sqrt{X}}\frac{\partial U}{\partial Z}=\frac{\partial^2 U}{\partial Z^2}-4\pi^2 U,\quad \frac{\partial V}{\partial X}+\frac{W_{00}}{\sqrt{X}}\frac{\partial V}{\partial Z}=-2\pi P+\frac{\partial^2 V}{\partial Z^2}-4\pi^2 V,$$

$$\frac{\partial W}{\partial X}-\frac{1}{2}\frac{W_{00}}{X^{3/2}}U+\frac{W_{00}}{\sqrt{X}}\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\frac{\partial^2 W}{\partial Z^2}-4\pi^2 W,\quad \frac{\partial U}{\partial X}+2\pi V+\frac{\partial W}{\partial Z}=0. \tag{2.4}$$

The initial conditions for $X = 0$ follow from (2.1):

$$U(0,Z)=0,\quad V(0,Z)=1,\quad W(0,Z)=-2\pi Z. \tag{2.5}$$

The boundary conditions for $Z \to 0$ are unknown *a priori* and should be found by matching with the solution in viscous subregion II′.

To find a solution of (2.4) satisfying (2.5), we assume that $U \equiv 0$. Then, eliminating the pressure from the second and third equations of (2.4) and using the continuity equation for expressing V in terms of W, we obtain the equation for the vertical component of velocity:

$$\frac{\partial B}{\partial X}+\frac{W_{00}}{\sqrt{X}}\frac{\partial B}{\partial Z}=\frac{\partial^2 B}{\partial Z^2}-4\pi^2 B,\qquad B=\frac{\partial^2 W}{\partial Z^2}-4\pi^2 W.$$

The solution of this equation satisfying (2.5) and the boundary condition on the wall $W(X,0) = G(X)$ specified by an arbitrary function $G(X)$ has the form

$$W=G(X)e^{-2\pi Z}-2\pi(Z-2W_{00}\sqrt{X})e^{-4\pi^2 X}. \tag{2.6}$$

The necessity of satisfying conditions (2.5) imposes the limitation $G(0) = 0$ on the function $G(X)$, which describes the displacing action of the boundary layer.

Having the expression for W, we can easily obtain a solution for V and P in subregion II″:

$$V=-G(X)e^{-2\pi Z}+e^{-4\pi^2 X},\qquad P=\Big(-\frac{1}{2\pi}\frac{dG}{dX}+\frac{W_{00}}{\sqrt{X}}\Big)e^{-2\pi Z}. \tag{2.7}$$

We now find the solution at the initial section of subregion II′. The boundary conditions at its external boundary follow from (2.6) and (2.7) and have the following form for $X \ll 1$:

$$Z/\sqrt{X}\to\infty:\quad U\to 0,\quad V\to 1,\quad W\to -2\pi(1+G(X))Z+4\pi W_{00}\sqrt{X}. \tag{2.8}$$

The solution for $X \ll 1$ in subregion II′ is sought in the form

$$U=2\pi X g'(\eta),\ \ V=l'(\eta),\ \ W=2\pi\sqrt{X}[(\eta/2)g'(\eta)-(3/2)g(\eta)-l(\eta)],\ \ P=O(1/\sqrt{X}), \tag{2.9}$$

where the functions g and l, which describe the profiles of perturbations of the streamwise and transverse components of velocity, depend on the self-similar variable $\eta = Z/\sqrt{X}$. Substitution of expressions (2.9) into the initial equations (2.3) and taking into account the terms of the least orders in X lead to boundary-value problems for ordinary differential equations for l and g:

$$l'''+(1/2)fl''=0,\qquad l(0)=l'(0)=0,\qquad l'(\infty)=1,$$

$$g''' + (1/2)fg'' - f'g' + (3/2)f''g = -f''l, \qquad g(0) = g'(0) = g(\infty) = 0.$$

We can easily see that $l(\eta) = f(\eta)$ and $g(\eta) = (\eta f' - f)/2$ are solutions of these problems. The solution obtained is illustrated in Fig. 2, which shows the profiles of perturbations of the streamwise u and transverse v components of velocity normalized to their maximum values.

The solution for inviscid subregion II$''$ contains an unknown function $G(X)$, which describes the displacing action of the boundary layer. Its form for $X \to 0$ is obtained by comparing the expression for W (2.8) with the asymptotic behavior of the solution for the vertical component of velocity in the boundary layer (2.9): $G(X) = -3\pi W_{00}\sqrt{X} + O(X)$ as $X \to 0$.

3. Solution in the Main Part of Region II. We seek the numerical solution of the complete system (2.3) for $X \approx 1$. These are equations of the parabolic type, and they require initial conditions in a certain cross section $X = X_0$ and boundary conditions on the wall and for $Z \to \infty$. For formulation of the boundary conditions, we need a solution uniformly suitable in terms of Z or a composite solution in the initial part of region II. It is found by a standard procedure [7] and coincides with (2.9) for U and V and has the following form for W:

$$W(X, Z) = 2\pi A\sqrt{X}[(\eta/2)g'(\eta) - (3/2)g(\eta) - f(\eta) - (3/2)W_{00}(e^{-2\pi Z} - 1) - 3\pi W_{00}Z]. \tag{3.1}$$

Since the complete system (2.3) is also valid in the inviscid part of the region considered, the boundary conditions for it as $Z \to \infty$ are taken as the corresponding limit of the solution of (2.6) and (2.7) in subregion II$''$:

$$Z \to \infty: \quad U(X, Z) \to 0, \quad V(X, Z) \to e^{-4\pi^2 X}, \quad W(X, Z) \to -2\pi(Z - 2W_{00}\sqrt{X})e^{-4\pi^2 X}.$$

These boundary conditions, the no-slip conditions on the wall $U(X, 0) = V(X, 0) = W(X, 0) = 0$, and the initial conditions (2.9) for U and V and (3.1) for W in the cross section $X = X_0$ form a complete statement of the problem for system (2.3). Note that this system does not contain any parameters; hence, the solutions of this problem U, V, W, and P are really universal functions of X and Z, and the form of the solution of (2.2) may be interpreted as the law of similarity.

To solve system (2.3), we eliminate the pressure by adding the derivative with respect to Z in the second equation with the third one multiplied by $4\pi^2$. In the resultant equation, we express the transverse component of velocity V in terms of U and W using the continuity equation. We replace the arising term $U_0\partial^2 U/\partial X^2$ containing the second derivative with respect to X by the expression found from the first equation of momentum differentiated with respect to X. As a result, we obtain the following equation for U and W:

$$U_0\frac{\partial B}{\partial X} + W_0\frac{\partial B}{\partial Z} - \frac{\partial U_0}{\partial X}B - \frac{\partial}{\partial X}\Big(\frac{\partial^2 U_0}{\partial Z^2}W\Big) - \frac{\partial^3 U_0}{\partial X\,\partial Z^2}W - 2\frac{\partial}{\partial Z}\Big(\frac{\partial U_0}{\partial X}\frac{\partial U}{\partial X}\Big)$$

$$- \frac{\partial W_0}{\partial X}\Big(\frac{\partial^2 U}{\partial Z^2} + 4\pi^2 U\Big) - \frac{\partial^3 U_0}{\partial X^2\,\partial Z}U = \frac{\partial^2 B}{\partial Z^2} - 4\pi^2 B, \qquad B = \frac{\partial^2 W}{\partial Z^2} - 4\pi^2 W.$$

This equation, the first equation of momentum, and the corresponding boundary and initial conditions form the problem for U and W, which was solved numerically using the marching method. The derivatives with respect to X were approximated by an implicit second-order difference scheme. The discretization of the equations in terms of Z was performed by the method of collocations, and the boundary conditions for $Z = 0$ and $Z \to \infty$ were satisfied by choosing appropriate basis functions.

4. Numerical Results and Analysis. To study the effect of the position of the initial cross section of the solution obtained, we calculated the evolution of perturbations for various values of X_0. It was found that the solution depends on X_0 for $X_0 \geqslant 10^{-4}$, and only for X_0 decreasing to approximately 10^{-5} do the results become independent of X_0 and remain constant with an error greater than 0.1%. The convergence of the solution with decreasing X_0 and the coincidence of the numerical solution with the analytical one (2.9) for $X_0 \leqslant 10^{-5}$ indicate that the formulation of the problem is not contradictory and evidence indirectly the credibility of the numerical method used.

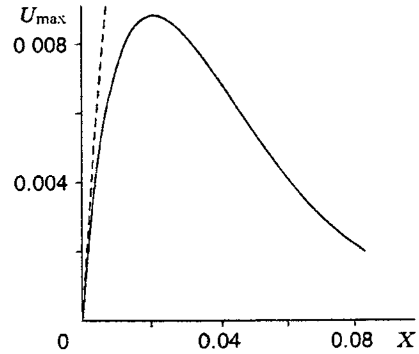

Fig. 3

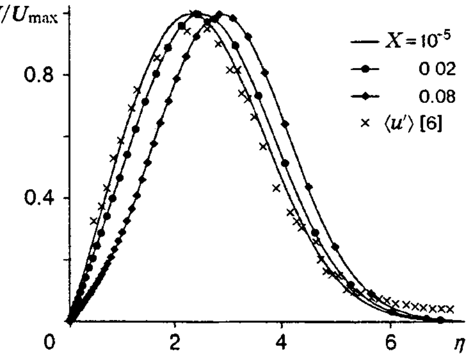

Fig. 4

The solution for U obtained for $X_0 \leqslant 10^{-5}$ is plotted in Figs. 3 and 4. The results for the velocity components V and W are of no interest, since they are negligibly small as compared to U for $X \approx 1$. The solid curve in Fig. 3 shows the dependence of the maximum (in terms of Z) perturbation $U_{\max}$ on X. For $X \approx 10^{-3}$, velocity perturbations increase almost linearly in accordance with the solution for $X \ll 1$ (2.7) for which the dependence of $U_{\max}$ on X is shown by the dashed curve. The perturbations reach a maximum at $X \simeq 0.02$ and then decay. A similar behavior of perturbations generated by streamwise vortices was obtained in [3, 4].

The profiles of the streamwise component of velocity perturbations along the vertical coordinate for $X = 10^{-5}$, 0.02, and 0.08 are plotted in Fig. 4. For convenience of comparison of the profiles with each other and with the experimental data of [6], they are plotted versus the coordinate $\eta = Z/\sqrt{X}$. It is seen from Fig. 4 that the maximum of velocity perturbations is gradually shifted away from the wall with distance from the leading edge. The change in the shape of the profile up to the cross section $X = 0.02$ corresponding to the maximum perturbations over the length is comparatively small, but it becomes significant at large distances from the leading edge where the perturbations decay. The profile of the low-frequency oscillations of velocity $\langle u' \rangle$ in the boundary layer in the case of an elevated level of free-stream turbulence, which was measured by Westin et al. [6], almost coincides with the velocity profiles of growing perturbations for $X = 10^{-5}$ and 0.02. The profile of velocity perturbations coinciding with the experimental one was also obtained in calculation of the evolution of perturbations generated by streamwise vortices in [3, 4]. This circumstance allows us to assume that the reason for increasing oscillations in the boundary layer may be both streamwise and perpendicular to the leading-edge vortices in the free stream.

To clarify the dependence of the solution obtained on the main parameters, we rewrite the expression for u (2.2) in the dimensional form:

$$u = (2\pi/a)\,\mathrm{Re}_b \ln(b'/\lambda')\, U((\delta'/\lambda')^2, z'/\lambda'). \tag{4.1}$$

Here $\delta' = \sqrt{\nu' x'/u'_\infty}$ is the boundary-layer thickness at a distance x' from the leading edge and $\mathrm{Re}_b = u'_\infty b'/\nu'$. It follows from Eq. (4.1) that the maximum value of perturbations in the boundary layer $u_{\max}$ (with accuracy to the logarithmic term) is independent of their transverse size λ' and is determined by the expression

$$u_{\max} \simeq (0.055/a)\,\mathrm{Re}_b \ln(b'/\lambda'). \tag{4.2}$$

The distance $x'_{\max}$ at which the maximum of perturbations is reached is proportional to their period squared:

$$x'_{\max} \simeq 0.02 u'_\infty \lambda'^2/\nu'. \tag{4.3}$$

It also follows from (4.1) that the transverse period of perturbations λ', which are amplified to the greatest extent by a given cross section x', is proportional (with accuracy to the logarithmic term) to the boundary-layer thickness in this cross section:

$$\lambda'_{\max} \simeq 7.07\delta'(1 + O(1/\ln(b'/\lambda'))). \tag{4.4}$$

The corollaries of the resultant solutions (4.3) and (4.4) correspond to the data of the experiment [6] and theory [3, 4] on amplification of quasi-steady vortices in the boundary layer on a flat plate with a sharp leading edge. In [6], the estimate of the period of the streaky structure has the form $\lambda' \simeq 9\delta'$, and in [4] the perturbations amplified to the greatest extent have the period $\lambda' \simeq 13\delta'$, which differs from (4.4) insignificantly. An expression for the distance $x'_{\max}$ similar to (4.3) is obtained from the results of [4].

Nevertheless, the conclusion that the maximum amplification of perturbations is independent of their period [see (4.2)] is significantly different from the known results for the boundary layer on a flat plate with a sharp leading edge. According to [4], we have $u_{\max} \sim \mathrm{Re} \sim u'_\infty \lambda'/\nu'$. A similar conclusion may be drawn from the experimental data of [6], if we take into account that the amplitude of velocity oscillations is $u_{\max} \sim \sqrt{\mathrm{Re}_x} \sim (u'_\infty x'/\nu')^{1/2}$ and their period is $\lambda' \sim \delta' \sim (\nu' x'/u'_\infty)^{1/2}$. The reason for this difference is the additional amplification of perturbations on the blunted leading edge owing to the deformation of the vorticity field in the flow past the leading edge. Indeed, in the case of interaction of flow nonuniformity with the leading edge, a transverse velocity arises at the edge of the boundary layer. This velocity exceeds the initial amplitude of nonuniformity by $A \approx (b'/\lambda') \ln (b'/\lambda')$ times. As a result, a perturbation with transverse and vertical components of velocity appears at the initial section of the boundary layer and above it. As is shown by Andersson et al. [4], the development of this kind of perturbations leads to their subsequent transformation into perturbations containing only the streamwise component of velocity, and the amplitude of the latter increases to a value which is greater than the initial one by Re times. A similar process of disturbance evolution in the boundary layer is described by the numerical solution in region II obtained in the present work. The product of the disturbance-amplification factors in the flow past the leading edge A and in the boundary layer Re yields the total amplification by $\mathrm{Re}_b \ln (b'/\lambda')$ times in accordance with (4.2). Since the amplification of perturbations at the leading edge is inversely proportional to their period and that in the boundary layer is directly proportional to the period, the total amplification is independent of the magnitude of perturbations.

The results obtained allow one to predict the special features of the laminar–turbulent transition on bodies with a blunted leading edge in the case of an elevated level of free-stream turbulence. The amplitude of oscillations in the boundary layer on such bodies should be almost constant over their length, but their transverse size, as on a flat plate with a sharp leading edge, should increase with distance from the leading edge. This character of disturbance evolution allows us to assume that flow turbulization on blunted bodies occurs either in the immediate vicinity of the leading edge or, if the level of flow turbulence is not sufficiently large, very far from the leading edge due to other mechanisms of disturbance growth. The amplification factor of vortex perturbations on blunted bodies is greater than on a flat plate with a sharp leading edge by a factor of b'/λ' or $\mathrm{Re}_b/\sqrt{\mathrm{Re}_x}$, and the transition on them should occur at lower free-stream turbulence than in experiments such as in [6]. Since the elements of constructions of flying vehicles (turbine blades, wings, fins) have generally blunted leading edges, this conclusion is important for determining the position of the laminar–turbulent transition in practice. For example, the amplification factor on a wing of a cargo plane with a leading-edge bluntness of $b' \approx 0.1$ m and a flight velocity of $u'_\infty \approx 200$ m/sec is roughly equal to 10^5 in accordance with (4.2), and the laminar–turbulent transition should be observed for an amplitude of nonuniformity of the flow profile of $\varepsilon \approx 10^{-6}$. Though the fraction of perturbations of the type of transverse nonuniformity of the velocity profile in an actual turbulent flow is unknown, we may assume that their characteristic amplitude ε varies between 0.1 and 0.01 of the turbulence level ε_{t}. Hence, the transition on the wing should occur at a level of turbulence $\varepsilon_{\mathrm{t}} \approx 0.01$–$0.001\%$, which is lower than in a low-turbulent wind tunnel and, possibly, corresponds to actual flight conditions.

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